Lecture 1

Topology 1

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Tue, Fridays — Lectures
Prob. Sessions — Tue.
Office Hours — 10 AM – 11 AM (Zoom)
Problem Sets — Tuesdays
Upload — moodle

Outline :- 1. Point set topology (1–3/4 week)

(German) Topologie – K. Jänich (online – HU Library)

Topology – K. Jänich (HU Library)

Algebraic Topology

Topology – James Munkres

2. Fundamental Groups and ways to compute them / Applications (4-9th week)

1. Topology – Munkres
2. Basic Topology – M. Armstrong (online, HU-Library)

Geometric

3. Algebraic Topology — Allen Hatcher.

3. Homology Theory

Alg. Top - Hatcher

Course Webpage — Lecture Notes, Problemsets (Moodle)

My webpage → Teaching → Topology I.

Lecture Notes — Chris Wendl.

Hype!

X set   additional structure

$\mathbb{R}$ 0 , $x+0 = x$
$x - x = 0$

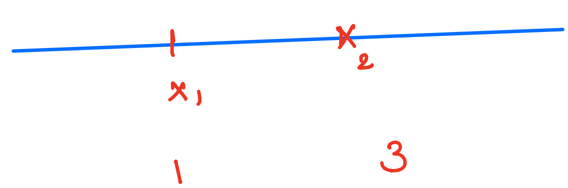
$\mathbb{R}^2$ $\{(a,b) \mid a,b \in \mathbb{R}\}$

$(a,b) + (c,d) = (a+c, b+d) \in \mathbb{R}^2$

$(0,0)$

$\vdots$

$\mathbb{R}^n$, $n \in \mathbb{N}$



$C([0,1])$

X sets   additional structure $\rightsquigarrow$ Topology

Classify "Topological spaces" upto some equivalence

↳ Homeomorphism

$\rightsquigarrow$ open sets , closed sets

$f : X \to Y$ "continuous"

Top. sp   Top. sp

Homeomorphism $f : X \to Y$ which is continuous, bijection (one-to-one, surjective /onto) and

$f^{-1} : Y \to X$ continuous.

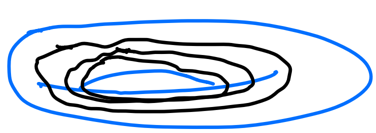
$X \cong Y$

homeomorphic

"intuitively"

Closed,

oriented surface of genus 3 $\ncong$



Torus

- surface of genus 1.

($\ncong$

genus 0.

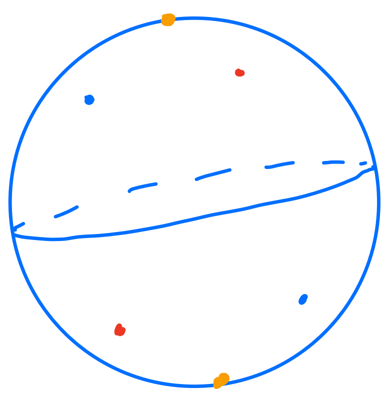
Not:- Σ_g - surface of genus $g \geq 0$.

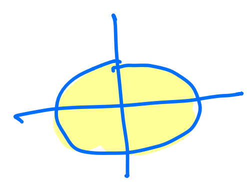
Ex. 1 Klein Bottle - surface which cannot be embedded in $\mathbb{R}^3$ (can be immersed)

" Topological space "

2. Real Projective Space $\mathbb{R}P^2$

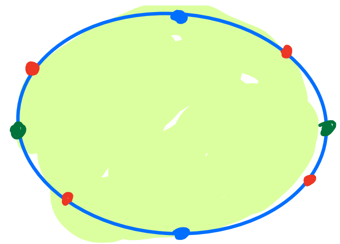
(i) $\mathbb{R}P^2 = S^2/\sim$ = set of equivalence classes w/ equivalence relation $x \sim -x$ where $x \in S^2 = \{ x \in \mathbb{R}^3 \mid |x| = 1 \}$



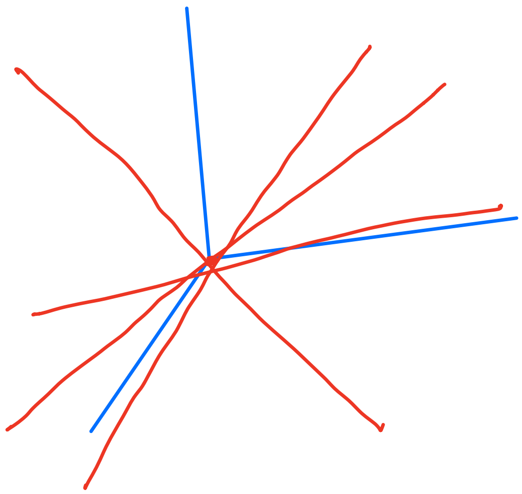


ii) $\mathbb{R}P^2 = D^2/\sim$ where $D^2 = \{ x \in \mathbb{R}^2 \mid |x| \leq 1 \}$

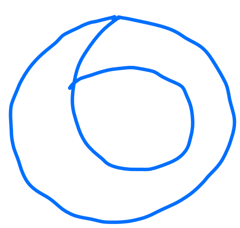
$\sim$ $z \sim -z$ where z is on the boundary of the disc.



iii) $\mathbb{RP}^2$ is the set of all lines through 0 in $\mathbb{R}^3$

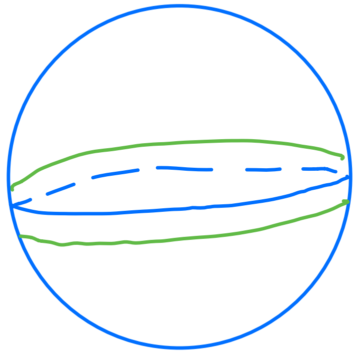


iv) Construct $\mathbb{RP}^2$ by glueing Möbius strip to a $\mathbb{D}^2$



nonorientability

$$M = \left\{ (\theta, t\cos(\pi\theta), t\sin(\pi\theta)) \in \mathbb{R}/\mathbb{Z} \times \mathbb{R}^2 \;\middle|\; \theta \in \mathbb{R}, t \in [-1,1] \right\}$$

$\mathbb{RP}^2 =$  $/\sim$

$n \geq 0$

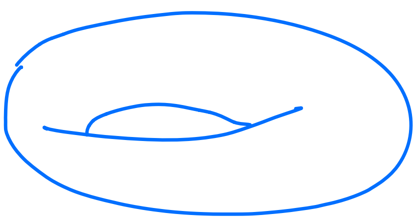
$S^n = \{ x \in \mathbb{R}^{n+1} \mid |x| = 1 \}$

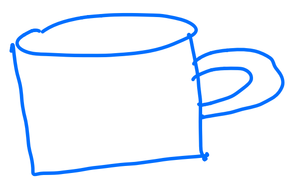
real projective n-space

$\mathbb{RP}^n = S^n / \{x \sim -x\} = \mathbb{D}^n / \{z \sim -z \text{ on } \partial \mathbb{D}^n\}$

boundary of $\mathbb{D}^n$.

$= \{$ lines of all lines through 0 in $\mathbb{R}^{n+1} \}$





Theorem (Later) A closed orientable surface Σ_g of genus g is homeomorphic to a closed orientable surface Σ_h of genus h $\Longleftrightarrow g = h$.

$f : \Sigma_g \to \Sigma_h$

$f^{-1} : \Sigma_h \to \Sigma_g$

$H(\Sigma_g) \not\cong H(\Sigma_h)$

If $g \neq h \Longrightarrow \Sigma_g \not\cong \Sigma_h$.

Algebraic Topology :- X (top. space)

$\rightsquigarrow$

algebraic object (group, ring)

$H(X)$

s.t. if $f : X \to Y$ continuous map

$\rightsquigarrow$

$f_* : H(X) \to H(Y)$

homomorphisms

f is homeomorphism

$\rightsquigarrow$

$f_* : H(X) \to H(Y)$ isomorphism

$f : X \to Y \xrightarrow[g]{} Z$

$g \circ f : X \to Z$

$\rightsquigarrow$

$(g \circ f)_* : H(X) \to H(Z)$

$= g_* \circ f_*$

Theorem (classification Theorem)

Every closed, connected and orientable surface S

then $S \cong \Sigma_g$, $g \geq 0$.

g must be unique.

Ques : (Poincaré Conjecture) 1904 – 100 years 2003

Suppose X is a closed, connected 3-manifold which is also "simply-connected". Then is

$X \cong S^3$?

Grigori Perelman

Millenenium Prize Problems

- metric spaces – Topological spaces

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