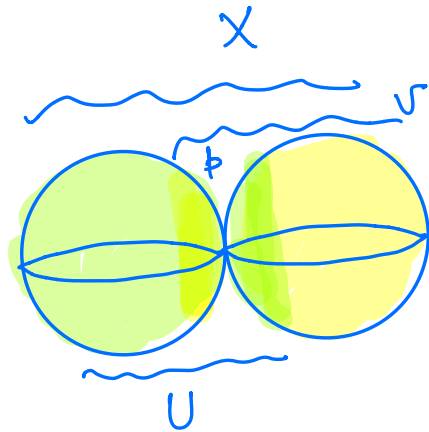


Problem Session 7

Pset 6

①

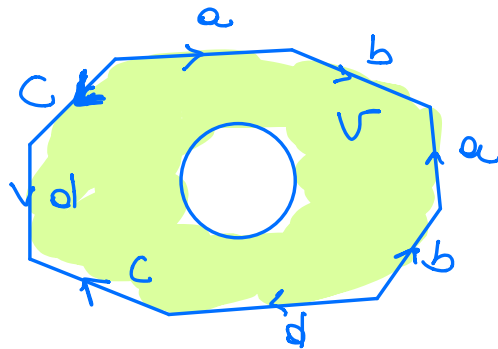
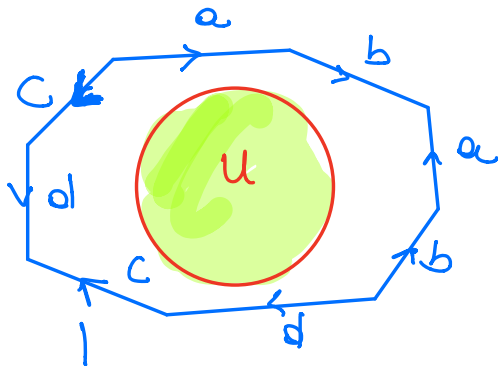
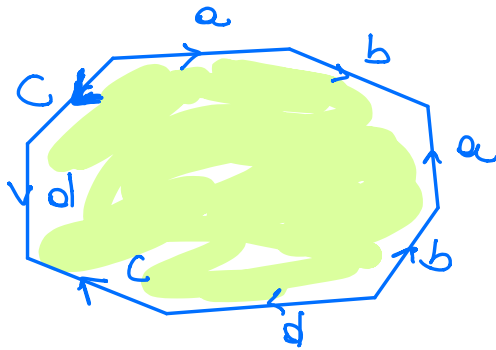
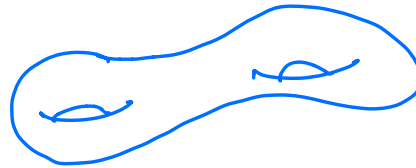


$\pi_1(X, p) = 0$
 U, V def. retracts to S^2
 $\Rightarrow \pi_1(U) \cong \pi_1(V) = 0$
 $U \cap V$ is a contractible space.

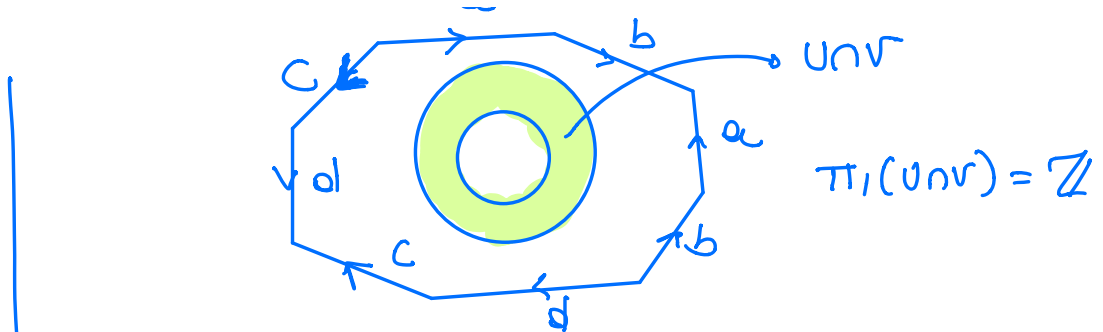
\therefore van Kampen thm \Rightarrow
 $\pi_1(X) = 0.$

②

$T \# T$

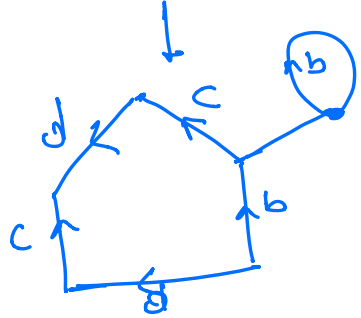
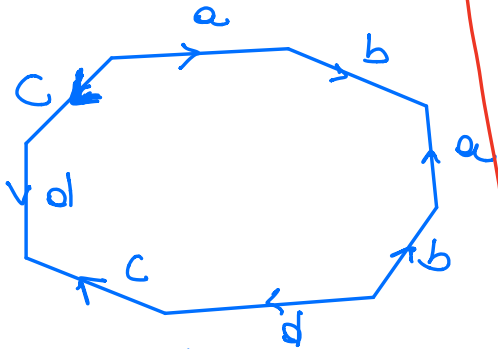


o.

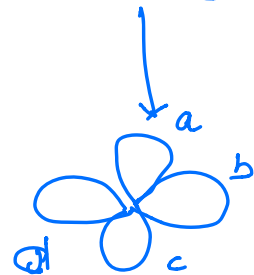
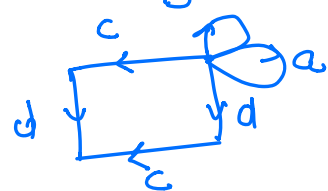


$\because U$ is contractible $\Rightarrow \pi_1(U) = \{0\}$.

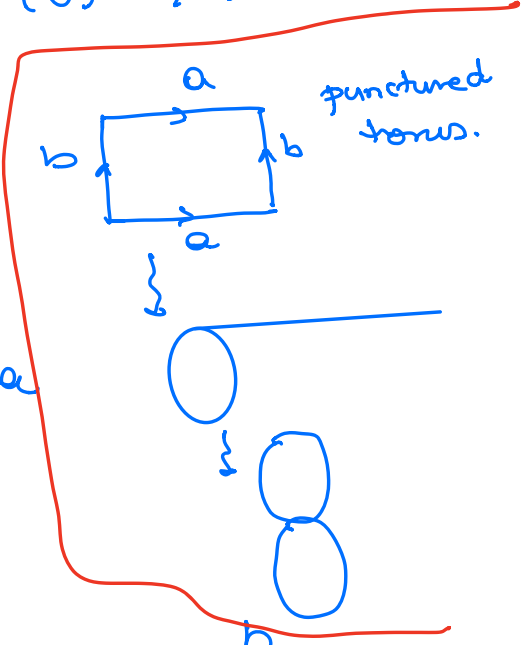
V deformation retracts

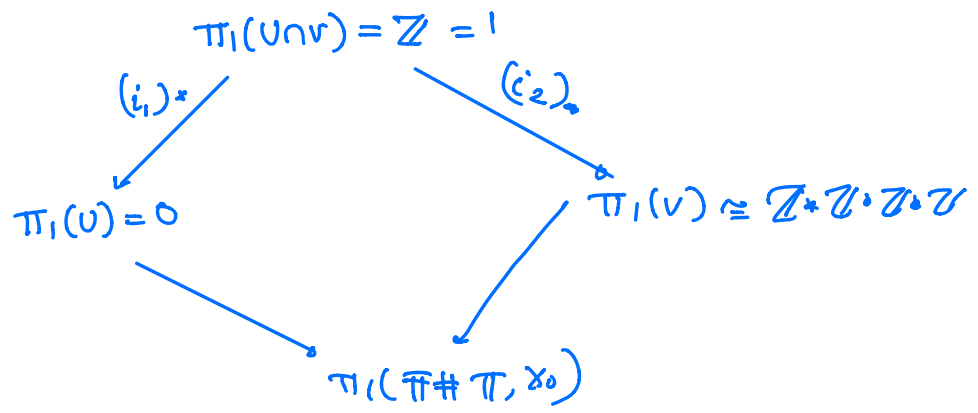


\rightsquigarrow



$\pi_1(V) \cong \mathbb{Z} * \mathbb{Z} * \mathbb{Z} * \mathbb{Z}$





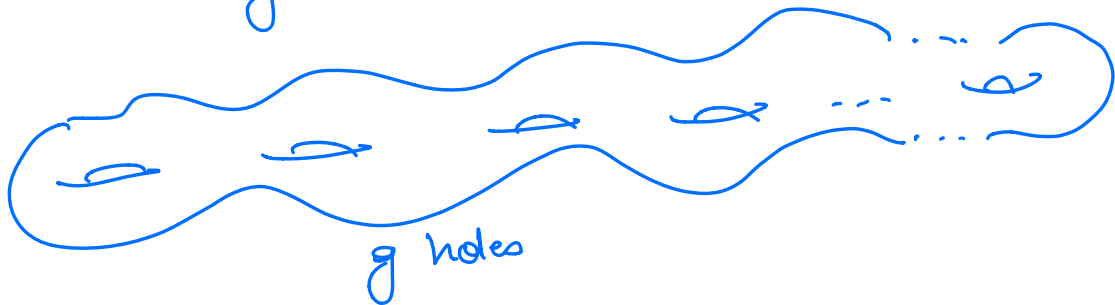
from the von Krompen thm.

$$\pi_1(T \# \tau) \cong \frac{\pi_1(U) * \pi_1(V)}{\mathcal{N}} \cong \frac{\mathbb{Z} * \mathbb{Z} * \mathbb{Z} * \mathbb{Z}}{\mathcal{N}}$$

loop $\gamma \rightsquigarrow 1 \in \mathbb{Z}$ correspond to $aba^{-1}b^{-1}cdc^{-1}d^{-1}$
 $U \cap V$

$$\pi_1(T \# \tau) = \frac{\mathbb{Z} * \mathbb{Z} * \mathbb{Z} * \mathbb{Z}}{\mathcal{N}} = \langle aba^{-1}b^{-1}cdc^{-1}d^{-1} \rangle$$

$$\underbrace{\tau \# \tau \# \tau \dots \# \tau}_g \quad \Sigma_g \rightsquigarrow 4g$$



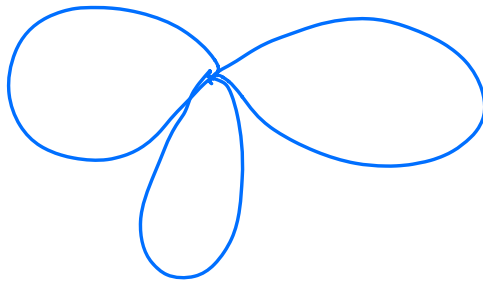
$$\pi_1(\Sigma_g) = \{ a_1, b_1, a_2, b_2, \dots, a_g, b_g \mid a_1 b_1 a_1^{-1} b_1^{-1} \dots a_g b_g a_g^{-1} b_g^{-1} = e \}$$

$G \xrightarrow{\text{abelianization}} G/[G, G]$

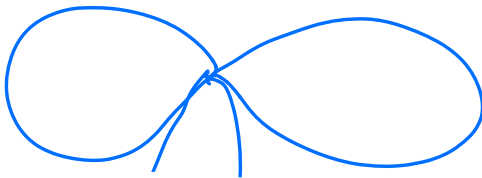
$$\underbrace{\mathbb{Z} \times \mathbb{Z} \times \dots \times \mathbb{Z}}_{2g}$$

Given any group G \exists top. space X s.t. $\pi_1(X) \cong G$.

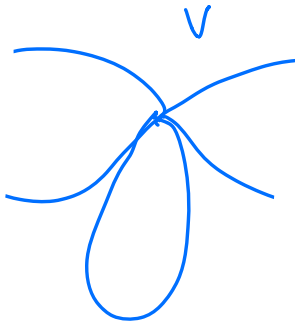
3



$s'v s'v s'v s'v$



u



v

\Rightarrow



$u \cap v$

x_0

$$\begin{aligned} \pi_1(u \cap v) &= 0 \\ \pi_1(v) &= \mathbb{Z} \\ \pi_1(u) &\cong \mathbb{Z} * \mathbb{Z} \end{aligned}$$

$$\pi_1(X) \cong \mathbb{Z} * \mathbb{Z} * \mathbb{Z}$$

for bouquet of n -circles $\rightarrow Y$

$$\pi_1(U) \cong \underbrace{\mathbb{Z} * \mathbb{Z} * \dots * \mathbb{Z}}_{(n-1)\text{ times}}$$

$$\pi_1(U) \cong \mathbb{Z}$$

$$\Rightarrow \pi_1(Y) \cong \underbrace{\mathbb{Z} * \mathbb{Z} * \dots * \mathbb{Z}}_{n\text{-times}}$$

④

