

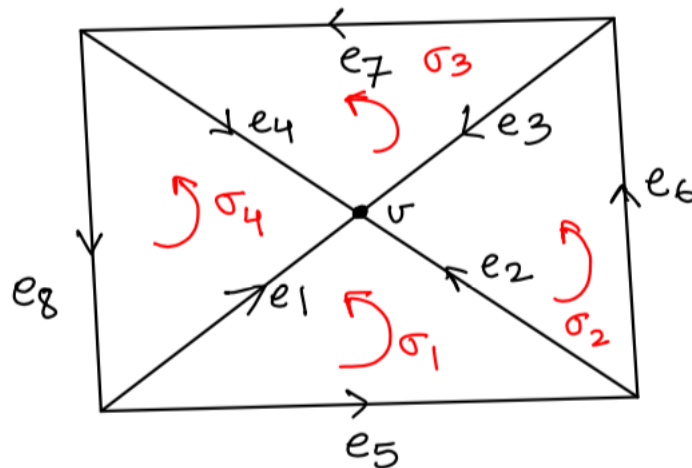
Problem Set 8
Due date: 13.07.2021

Instructions

Problems marked with (*) will be graded. Solutions may be written up in German or English (preferable) and should be handed in before the Problem sessions on the due date. For problems without (*), you do not need to write up your solutions, but it is highly recommended that you think through them before the next Tuesday lecture.

Problems

- (1) (*) Showing all the details, compute all the homology groups of the sphere S^2 , i.e., compute $H_n(S^2)$, $n \geq 0$.
- (2) (*) Showing all the details, compute all the homology groups of the complex in the figure.



- (3) (*) The **Klein bottle** is a quotient space which can be described as a quotient of the square $[0, 1] \times [0, 1]$ with sides identified by the relations:

$$(0, y) \sim (1, y) \quad 0 \leq y \leq 1 \quad \text{and} \quad (x, 0) \sim (1 - x, 1) \quad 0 \leq x \leq 1.$$

Draw a simplicial complex representation of the Klein bottle (no details required) and compute all its homology groups, showing all the details.