**Problem Set 6**
**Due date: 22.06.2021**

**Instructions**

Problems marked with $(*)$ will be graded. Solutions may be written up in German or English (preferable) and should be handed in before the Problem sessions on the due date. For problems without $(*)$, you do not need to write up your solutions, but it is highly recommended that you think through them before the next Tuesday lecture.

**Problems**

(1) Let $X$ be the union of two copies of $S^2$ having a point in common. Compute its fundamental group and show all the details.

(2) $(*)$ Compute showing details, the fundamental group of the double torus $T\#T$. Generalize this to the $n-$holed torus $\underbrace{T\#T\#\cdots\#T}_{\text{n-times}}$.

(3) $(*)$ Compute showing details, the fundamental group of the bouquet of three circles, i.e., three circles all having a point in common. Generalize the result to the fundamental group of bouquet of $n$ circles.

(4) $(*)$ Compute the fundamental group of the "dunce cap" which is the quotient space of a triangle by associating the three sides with each other as shown below.