Ricci Flow II
Universität Hamburg, Summer Semester 2025

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Website:	https://s-dwivedi.github.io/RicciFlowSS25.html
Lectures:	Thursdays 12:15 - 13:45 in Geom H3
Problem Sessions:	every alternate Thursdays starting 17th April 2025 14:15 - 15:45 in Sed 19, 205
Language:	English

Short Description

Right from its introduction by Hamilton in 1982, the Ricci flow has found applications in both geometry and topology. Perhaps the crowning achievement of the Ricci flow is the proof of the Poincaré conjecture or more generally the proof of the Thurston's Geometrization conjecture by Perelman. This course intends to be a continuation of the introductory course on the Ricci flow which was offered in the last semester. The target audience is Bachelors and Masters's students and PhD students. It is expected that either you have attended the Ricci flow course in WS 2024-25 or have a solid understanding of Riemannian geometry and analysis (especially PDEs). A detailed (preliminary) discussion of topics is outlined below.

Topics to be covered (the topics marked with * might be covered if there is interest among the participants.)

(1) Introduction to the Ricci flow.
(2) Evolution equations of intrinsic geometric quantities along the flow.
(3) Uhlenbeck's trick: evolution of the Riemann curvature tensor; Hamilton's theorem on positivity of Riemann curvature being preserved.
(4) Curvature estimates and long time existence, in particular, Hamilton's proof of the Poincaré conjecture for 3-manifolds admitting a metric of positive Ricci curvature.
(5) Vector bundle maximum principles; curvature pinching estimates and Hamilton–Ivey pinching estimate.
(6) Li–Yau Harnack inequality and Hamilton's Harnack estimates for the Ricci flow. (Chow–Chu's approach to Hamilton's Harnack estimates using the space-time approach*)
(7) Ricci solitons item Ricci flow as a gradient flow: Perelman's $\mathcal{F}$ and $\mathcal{W}$ functionals and their monotonicity.
(8) Perelman's No Local Collapsing theorem (proof of Hamilton's little loop conjecture.)
(9) Logarithmic Sobolev inequalities.
(10) Overall idea of Perelman's proof of the Thurston's Geometrization Conjecture.

The aforementioned topics are much more than what we'll actually be able to cover in the course.

Grading

The grades in the course will be decided when the classes start:

Literature

There are excellent introductions to the subject of the Ricci flow and the materials presented in the class will be followed from the references mentioned below. In particular, [CK04], [CLN06] and [Top06] are good sources for self-study as well.

References

[CC95] Bennett Chow and Sun-Chin Chu, *A geometric interpretation of Hamilton's Harnack inequality for the Ricci flow*, Math. Res. Lett. **2** (1995), no. 6, 701–718. MR1362964 ↑

[CCG$^+$07] Bennett Chow, Sun-Chin Chu, David Glickenstein, Christine Guenther, James Isenberg, Tom Ivey, Dan Knopf, Peng Lu, Feng Luo, and Lei Ni, *The Ricci flow: techniques and applications. Part I*, Mathematical Surveys and Monographs, vol. 135, American Mathematical Society, Providence, RI, 2007. Geometric aspects. MR2302600 ↑

[CK04] Bennett Chow and Dan Knopf, *The Ricci flow: an introduction*, Mathematical Surveys and Monographs, vol. 110, American Mathematical Society, Providence, RI, 2004. MR2061425 ↑2

[CLN06] Bennett Chow, Peng Lu, and Lei Ni, *Hamilton's Ricci flow*, Graduate Studies in Mathematics, vol. 77, American Mathematical Society, Providence, RI; Science Press Beijing, New York, 2006. MR2274812 ↑2

[CZ06] Huai-Dong Cao and Xi-Ping Zhu, *A complete proof of the Poincaré and geometrization conjectures—application of the Hamilton-Perelman theory of the Ricci flow*, Asian J. Math. **10** (2006), no. 2, 165–492. MR2233789 ↑

[Ham82] Richard S. Hamilton, *Three-manifolds with positive Ricci curvature*, J. Differential Geometry **17** (1982), no. 2, 255–306. MR664497 ↑

[Ham86] ———, *Four-manifolds with positive curvature operator*, J. Differential Geom. **24** (1986), no. 2, 153–179. MR862046 ↑

[Ham93] ———, *The Harnack estimate for the Ricci flow*, J. Differential Geom. **37** (1993), no. 1, 225–243. MR1198607 ↑

[Ham95a] ———, *A compactness property for solutions of the Ricci flow*, Amer. J. Math. **117** (1995), no. 3, 545–572. MR1333936 ↑

[Ham95b] ———, *The formation of singularities in the Ricci flow*, Surveys in differential geometry, Vol. II (Cambridge, MA, 1993), 1995, pp. 7–136. MR1375255 ↑

[KL08] Bruce Kleiner and John Lott, *Notes on Perelman's papers*, Geom. Topol. **12** (2008), no. 5, 2587–2855. MR2460872 ↑

[MT07] John Morgan and Gang Tian, *Ricci flow and the Poincaré conjecture*, Clay Mathematics Monographs, vol. 3, American Mathematical Society, Providence, RI; Clay Mathematics Institute, Cambridge, MA, 2007. MR2334563 ↑

[Per02] Grisha Perelman, *The entropy formula for the Ricci flow and its geometric applications*, arXiv Mathematics e-prints (November 2002), math/0211159, available at `math/0211159`. ↑

[Top06] Peter Topping, *Lectures on the Ricci flow*, London Mathematical Society Lecture Note Series, vol. 325, Cambridge University Press, Cambridge, 2006. MR2265040 ↑2