Exponential functions

An <u>exponential function</u> with base a has the form

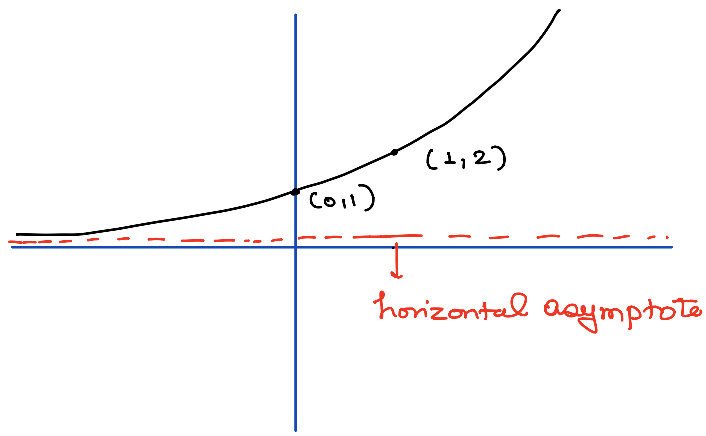
$$f(x) = a^x$$

where $a > 0$, $a \neq 1$ (otherwise $f(x) = 1$)

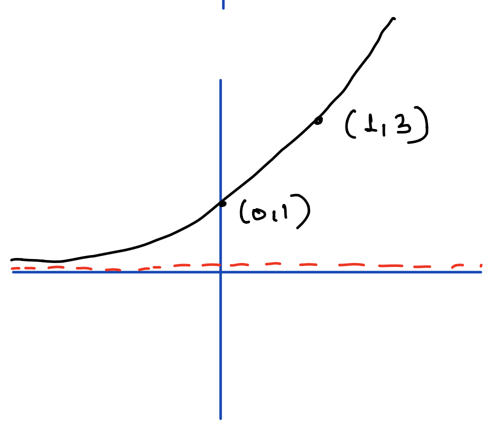
Any exponential function has

- Domain — $\mathbb{R}$
- Range — $(0, \infty)$
- horizontal asymptote at $y = 0$ (x-axis)

<u>e.g.</u> $f(x) = 2^x$

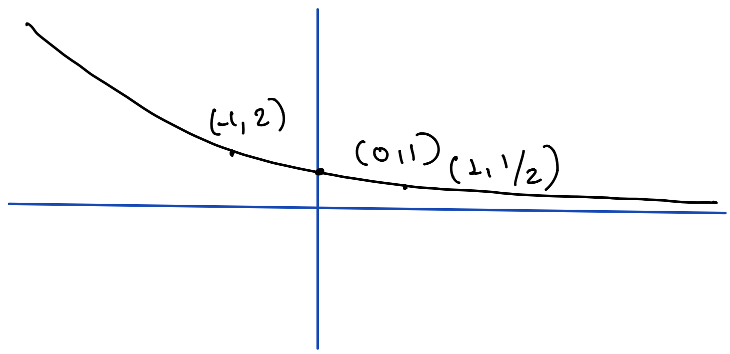


$f(x) = 3^x$



What about $f(x) = \left(\frac{1}{2}\right)^x$?

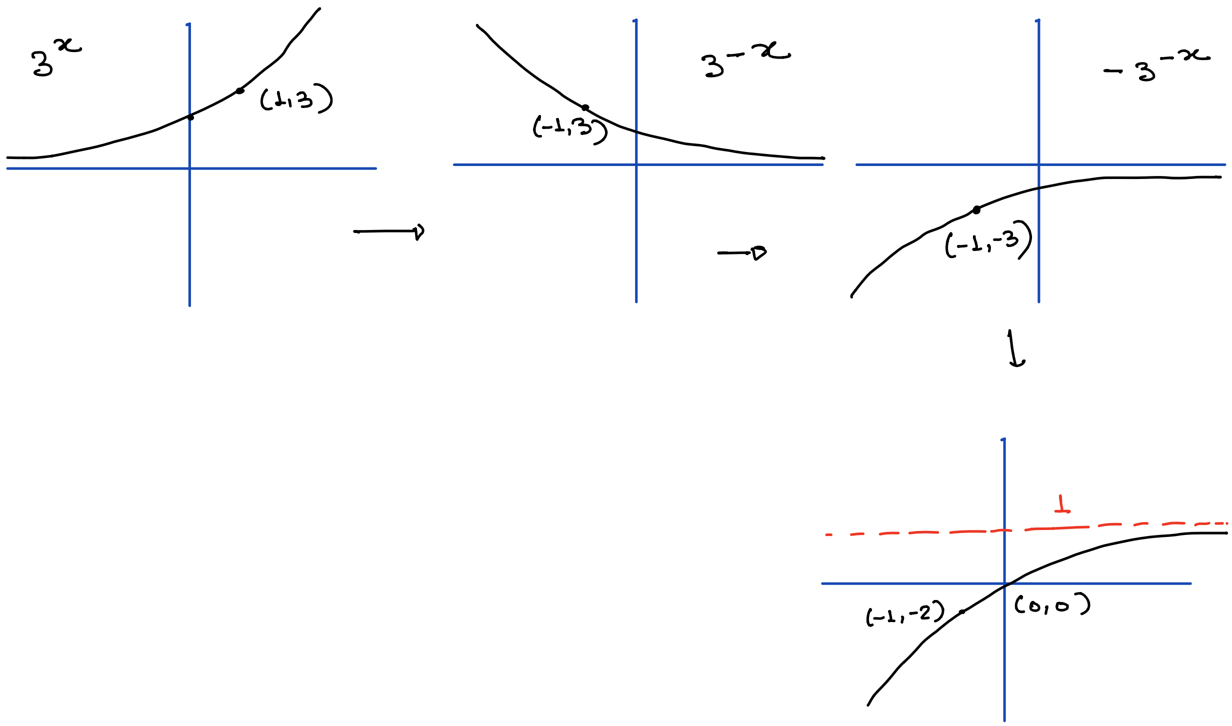
note that $\left(\frac{1}{2}\right)^x = (2^{-1})^x = 2^{-x}$. So, from what we have learned in the section about reflection, the graph of $f(x) = \left(\frac{1}{2}\right)^x$ will be a reflection of $y = 2^x$ along the y-axis.



Similarly, $f(x) = -3^x$ will have a graph which is a reflection of $f(x) = 3^x$ along the x-axis.

E.g. Sketch the graph of $f(x) = -3^{-x} + 1$.

Sol^n We'll first have to reflect the graph of 3^x along the y-axis to get 3^{-x}, then reflect the result along the x-axis to get -3^{-x} and finally an upward shift by 1 unit to get the graph of $-3^{-x}+1$.



Solving Exponential Equations

When solving equations involving exponential functions, remember the important rule :-

$$\boxed{\text{For } a > 0,\ a \neq 1,\ a^x = a^y \iff x = y}$$

* bases must be the same.

E.g. Solve the following :-

a) $3^{3x} = 3^5$ b) $3^{3x} = 9^5$

c) $4^{x^2+2} = 8^{x^2}$ d) $\left(\frac{1}{7}\right)^x = (49)^{3x+1}$

Sol^n a) $3^{3x} = 3^5 \Rightarrow 3x = 5 \Rightarrow x = \frac{5}{3}$

b) $3^{3x} = 9^5$. The bases are different right now. However, observe $9 = 3^2$ and recalling the properties of exponents, we get

$$3^{3x} = (3^2)^5 = 3^{10} \Rightarrow 3x = 10 \Rightarrow x = \frac{10}{3}.$$

c) $4^{x^2+2} = 8^{x^2} \Rightarrow (2^2)^{x^2+2} = (2^3)^{x^2}$

$$\Rightarrow 2^{2x^2+4} = 2^{3x^2}$$

$$\Rightarrow 2x^2+4 = 3x^2$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = +2 \text{ or } x = -2.$$

d) $\left(\frac{1}{7}\right)^x = (49)^{3x+1} \Rightarrow (7^{-1})^x = (7^2)^{3x+1}$

$$\Rightarrow 7^{-x} = 7^{6x+2}$$

$$\Rightarrow -x = 6x+2 \Rightarrow 7x = -2$$

$$\Rightarrow x = -\frac{2}{7}.$$

Note that the bases must be same for solving $a^x = a^y$.

Thus, for example, $3^x = 5$ cannot be solved using this method. So we need better techniques [Logarithms]

The number "e"

Def^n e is a constant ($e \simeq 2.718281\ldots$) such that the graph of e^x has slope 1 at $x=0$. [Very important ie next chapter].

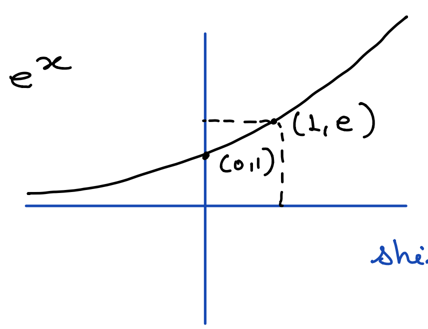
or

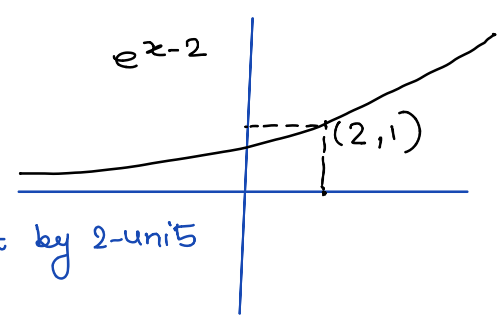
$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n \quad \text{(next chapter)}$$

e is an extremely important constant w/ applications in maths, biology, finance etc.

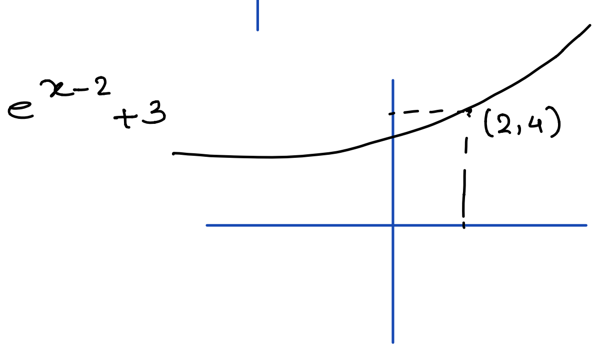
e.g. Sketch the graph of $e^{x-2} + 3$.

Sol





shift right by 2-units



move upwards by 3-units