Lecture 7

Recall from Lecture 1 that

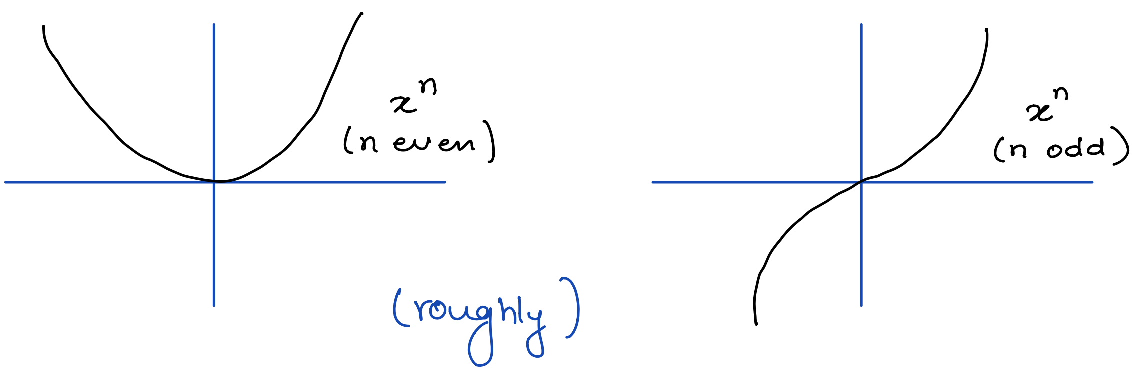
A polynomial of degree n is a function of the form

$$y = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

Here $a_i \in \mathbb{R}$, $i = 0, \ldots, n$, $a_n \neq 0$.

Domain $= \mathbb{R}$

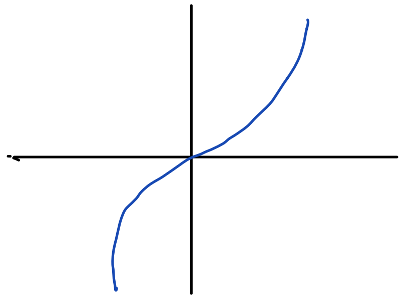
Range $= \mathbb{R}$ <u>if degree n is odd</u>



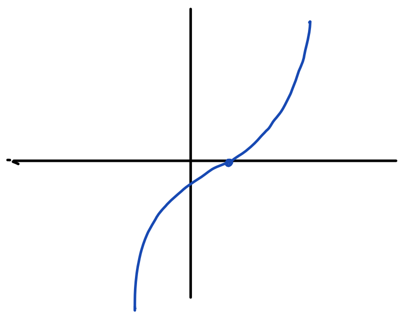
We can use the techniques from Lecture 6 to sketch the graph of polynomials.

<u>e.g.</u> Sketch $y = -(x-2)^3 + 2$.

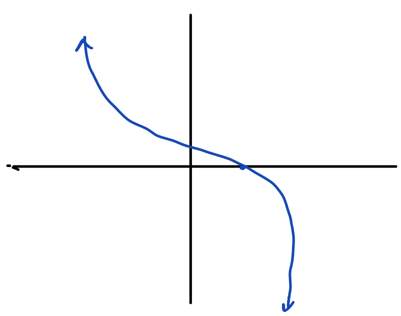
<u>soln.</u> We start with the graph of x^3.



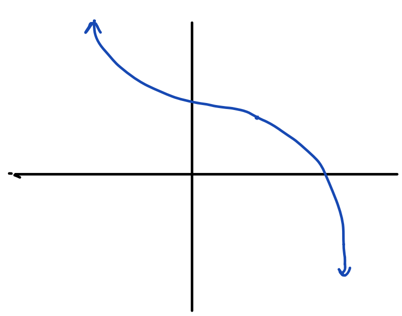
$\therefore$ $(x-2)^3$ will shift the graph by 2 units towards right.



$-(x-2)^3$ will reflect the above graph along the x-axis.



finally $-(x-2)^3+2$ will move the above graph upwards by 2 units.



note that the x-intercept $(y=0)$ is

$$0 = -(x-2)^3+2 \Rightarrow (x-2)^3 = 2 \Rightarrow x = \sqrt[3]{2}+2$$
$$\simeq 3.26$$

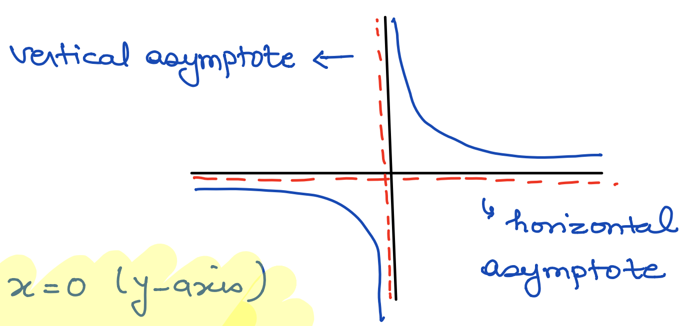
and y-intercept $(x=0)$ is $y = -(0-2)^3+2 = 10$.

Recall from Lecture 2 that

A rational function is of the form $\dfrac{p(x)}{q(x)}$

where $p(x)$ and $q(x)$ are polynomials, $q(x) \neq 0$.
The points where $q(x)=0$ are not in the domain. At these points, f has a vertical asymptote.

e.g. $y = \dfrac{1}{x}$ has domain $(-\infty,0)\cup(0,\infty)$



Thus, $y=\dfrac{1}{x}$ has a
vertical asymptote at $x=0$ (y-axis)
horizontal asymptote at $y=0$ (x-axis)

For a rational function, $\dfrac{p(x)}{q(x)}$, if $p(a)\neq 0$ and $q(a)=0$ then $\dfrac{p(x)}{q(x)}$ has a vertical asymptote at $x=a$.

Remark :- Finding horizontal asymptote requires the knowledge of limits, which we will do later.

e.g. For $y = \dfrac{x^2-3x+2}{x^2-3x}$ find :- a) vertical asymptotes

b) x-intercepts

c) y-intercept.

<u>solution</u> $p(x) = x^2 - 3x + 2 = (x-2)(x-1)$

$q(x) = x^2 - 3x = x(x-3)$

a) For vertical asymptotes, we need to solve $q(x) = 0$

$\Rightarrow \quad x(x-3) = 0 \quad \Rightarrow \quad x = 0$ and $x = 3$.

Also, $p(0) = 2 \neq 0$ and $p(3) = 2 \neq 0$

$\therefore$ both $x = 0$ and $x = 3$ are vertical asymptotes.

b) For finding x-intercepts, we set $y = 0$

$\Rightarrow \quad \dfrac{x^2-3x+2}{x^2-3x} = 0 \quad \Rightarrow \quad x^2 - 3x + 2 = 0$

$\Rightarrow \quad (x-2)(x-1) = 0 \quad \Rightarrow \quad x = 1$ or $x = 2$

Thus the x-intercepts are $(1,0)$ and $(2,0)$.

c) For finding y-intercepts, we put $x = 0$. However, $x = 0$ is not in the domain. Thus, no y-intercepts.

E.g. Give an example of a rational function with
i) vertical asymptotes at $x = 5$ and $x = 10$ and
ii) y-intercept at $(0,1)$.

Solution: Since we must have vertical asymptotes at $x = 5$ and $x = 10 \Rightarrow$ in the denominator, we must have $(x-5)(x-10)$.
Now y-intercept as $(0,7)$ means that when we put $x = 0$, we must get 7. There are many such functions, e.g.

$$f(x) = \frac{50}{(x-5)(x-10)} \quad \text{or} \quad f(x) = \frac{x}{(x-5)(x-10)} + 7$$

or

$$f(x) = \frac{x^2 + 3x + 50}{(x-5)(x-10)}$$

Thus, there are many solutions. In such cases, writing any one solution is sufficient.

o ———— x ———— x ———— o