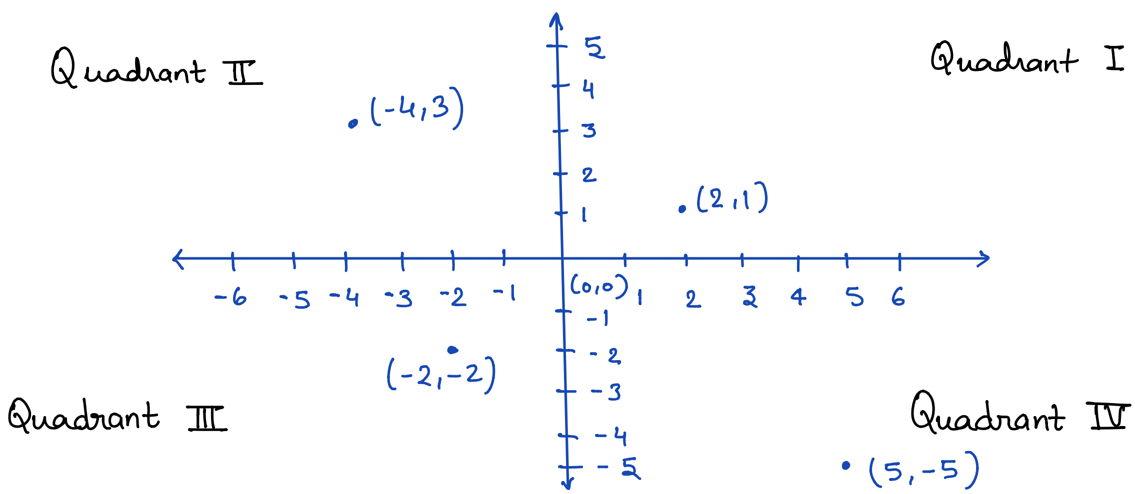
Lecture 4

Lines & Linear functions

We'll work w/ cartesian coordinates (x, y) :

x - coordinate : how far left/right

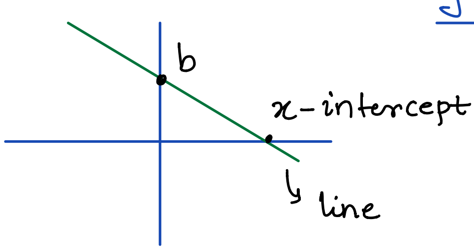
y - coordinate : how far up/down



Lines A line has an equation $y = mx + b$.

$m =$ slope of the line

$b =$ y-intercept (where the line crosses the y-axis)

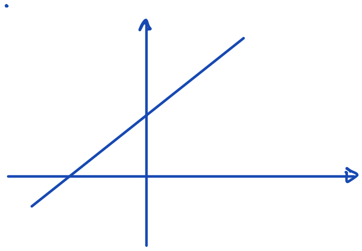


$$\text{Slope} = \frac{\text{"rise"}}{\text{"run"}} = \frac{\text{change in } y}{\text{change in } x}$$

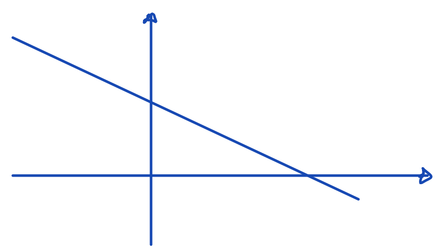
Given two points (x_1, y_1), (x_2, y_2) on the line, we can calculate the slope of the line as

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

e.g.



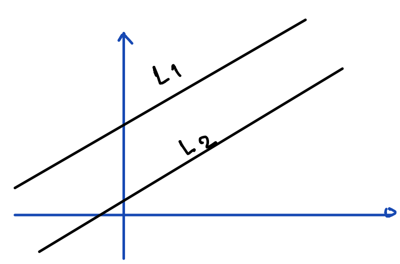
Line w/ positive slope
$(m > 0)$



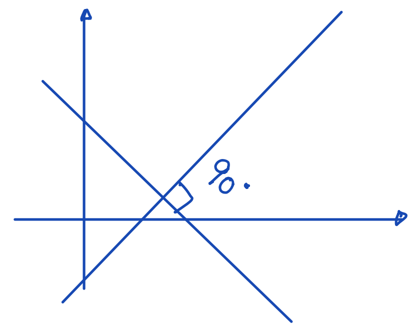
Line w/ negative slope
$(m < 0)$

Some Facts

- Horizontal lines have slope $m = 0$.
- Vertical lines have "infinite" slope or undefined slope $(m = \infty)$.
- Two lines are parallel if they have the same slope.



- Two lines are perpendicular if one has slope m and the other has slope $-\frac{1}{m}$. The lines meet at 90°.



Following are the types of questions which we can ask about lines:-

1) Find equation of a line through 2 given points.

2) Find equation of a line w/ slope m and passing through a given point (x_0, y_0).

3) Find equation of a line through a given point (x_0, y_0) which is parallel or perpendicular to another line.

Let's see how to tackle each of the above questions.

E.g. 1) Find the equation of a line through $(1,1)$ and $(3,-4)$.

solution note that the slope $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-4-1}{3-1}$

$$= -\frac{5}{2}$$

Thus we know that $y = -\frac{5}{2}x + b$.

To find b, substitute the x and y values of any of the given points. Putting $x = 1 = y$ gives

$1 = -\frac{5}{2} + b \Rightarrow b = \frac{7}{2}$. So the equation of the line is $\boxed{y = -\frac{5}{2}x + \frac{7}{2}}$.

2) Find the equation of the line through (1,3) and parallel to $y = 3x+5$.

Solution: note that parallel lines have the same slope $\Rightarrow m = 3$. Thus $y = 3x+b$. To find b,

sub. $x = 1$, $y = 3 \Rightarrow 3 = 3.1 + b \Rightarrow b = 0$.

Thus, the equation of the line is $y = 3x$.

3) Find the equation of the line through (1,3) and perpendicular to $3y - 5x = 1$.

Solution: Note that $3y - 5x = 1$ doesn't have the slope 5 as it is not yet in the form $y = mx+b$.

We have $y = \frac{5}{3}x + \frac{1}{3} \Rightarrow$ slope $= \frac{5}{3}$

now slope of the perpendicular line is $\dfrac{-1}{\frac{5}{3}} = \dfrac{-3}{5}$

$\Rightarrow \quad y = -\frac{3}{5}x + b$. now sub. $x = 1$, $y = 3$ to get

$b = \frac{18}{5}$ and so the equation of the line is

$y = -\frac{3}{5}x + \frac{18}{5}$.

Graphing Lines

To graph a line, just plot two points and connect them.

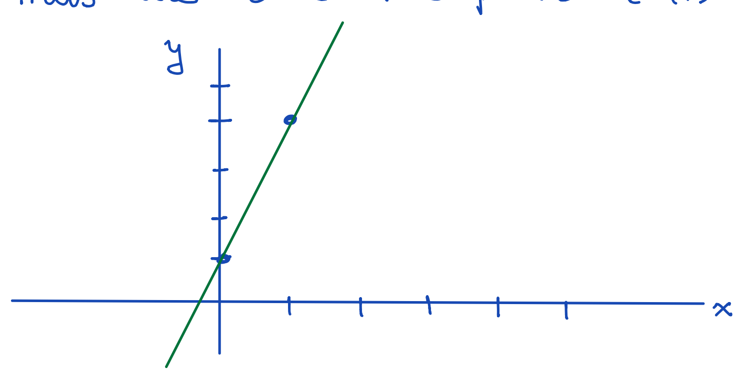
e.g. Graph $y = 3x + 1$.

Solution Note that the y-intercept $= 1$

$\Rightarrow$ the line goes through $(0,1)$.

Now plug any value of x, say $x = 1$ to get

$y = 4$. Thus we have two points : $(0,1)$ and $(1,4)$.



Determining where two lines meet

If two lines meet at a point then their y-values should be the same. Hence we just equate the y values and solve for x. Once we find x, we find y by putting the x-value back in the equation.

e.g. Where do $y = x+1$ and $y = -x-1$ intersect?

solution we have $x+1 = -x-1$

$$\Rightarrow 2x = -2 \Rightarrow x = -1$$

now $x = -1 \Rightarrow y = -1+1 \Rightarrow y = 0$. Thus the point of intersection is $(-1, 0)$.

We'll start with functions now.

Now, we will learn about functions.

Definition A function is a rule that assigns to each element of one set exactly one element from another set.

notation $y = f(x)$

x = independent variable

y = dependent variable

The domain of a function is the set of all possible values that x can take. [input]

The range of a function is the set of all possible values that y can take. [output]

E.g.

Function	Domain	Range
$y = x^2$	$\mathbb{R}$ (all real no.s) or $(-\infty, \infty)$	$[0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$ (can't take $\sqrt{\cdot}$ of negative)	$[0, \infty)$
$y = \sin(x)$	$\mathbb{R}$	$[-1, 1]$