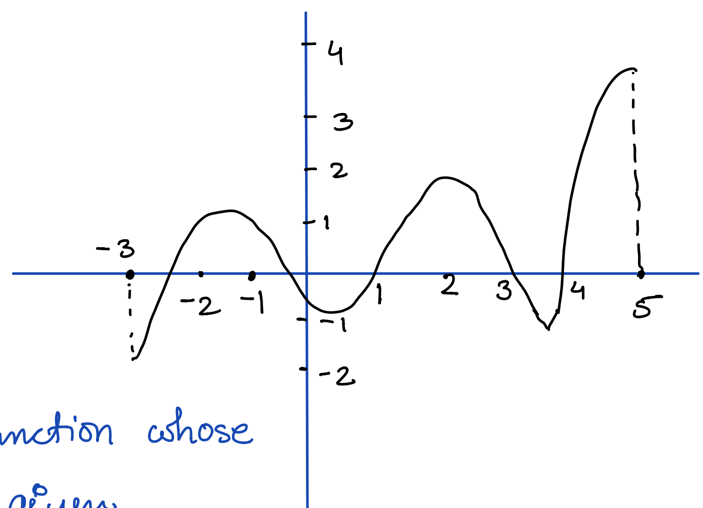
Lecture 25

In the last lecture, we learnt about local max/min.
In this lecture we'll see global max/min.

Def^n A point c in the domain of $f(x)$ is called a

- global max if $f(c) \geq f(x)$ for all x in the domain.
- global min if $f(c) \leq f(x)$ for all x in the domain.

e.g.



for the function whose graph is given

global max is at $x = 5$ w/ the value 4

global min is at $x = -3$ w/ the value -2

Global extrema (i.e., global max and min) will always be calculated on a closed interval $[a,b]$. To find them.

1. find all critical points inside $[a,b]$.
2. compute $f(a)$, $f(b)$ and $f(\text{critical points})$
3. Global max = biggest value in 2
   Global min = biggest value in 1

Ques. find the global extrema for $f(x) = x^2 - 2x$ in $[-2, 2]$.

Sol We first find all the critical points.

$$f'(x) = 2x - 2 = 0 \implies x = 1$$

next we compute
$$f(-2) = (-2)^2 - 2(-2) = 4+4 = 8$$
$$f(2) = 2^2 - 2 \cdot 2 = 0$$
$$f(1) = 1^2 - 2 \cdot 1 = -1$$

$\therefore$ global max = 8 at $x = -2$
global min = -1 at $x = 1$.

Ques find the global extrema for the following.

1. $f(x) = 2x^3 - 9x^2 + 12x + 6$ in $[0, 2]$
2. $f(x) = 6x^{2/3} - 4x + 2$ in $[-1, 1/2]$

$\underline{\text{Sol}^n}$ ① $f'(x) = 6x^2 - 18x + 12 = 6(x^2-3x+2) = 0$

$\Rightarrow 6(x-2)(x-1) = 0$

$\Rightarrow x=2$ and $x=1$ are the critical points inside $[0,2]$.

We compute $f(0) = 6$

$f(2) = 2.(2)^3 - 9.2^2 + 12.2 + 6$

$= 16 - 36 + 24 + 6$

$= 10$

$f(1) = 2 - 9 + 12 + 6 = 10$

Thus global max $= 10$ at $x = 1$ and 2

global min $= 6$ at $x = 0$.

② $f'(x) = 6.\frac{2}{3} x^{\frac{2}{3}-1} - 4 = 0$

$\Rightarrow 4x^{-\frac{1}{3}} - 4 = 0 \Rightarrow 4x^{-1/3} = 4$

$\Rightarrow x = 1$

Note that $f'(x)$ DNE at $x=0$. However $x=1$ doesn't lie in $[-1, 1/2]$ so we ignore this and hence the only critical point is $x=0$.

We compute $f(-1) = 6.(-1)^{2/3} - 4(-1) + 2$

$$= 6 + 4 + 2 = 12$$

$$f(0) = 2$$

$$f(1/2) = 6.\left(\frac{1}{2}\right)^{\frac{2}{3}} - 4.\frac{1}{2} + 2$$

$$= 6.\left(\frac{1}{4}\right)^{\frac{1}{3}} - 2 + 2 \approx 3.78$$

Thus, global max $= 12$ at $x = -1$

global min $= 2$ at $x = 0$.