Lecture 24

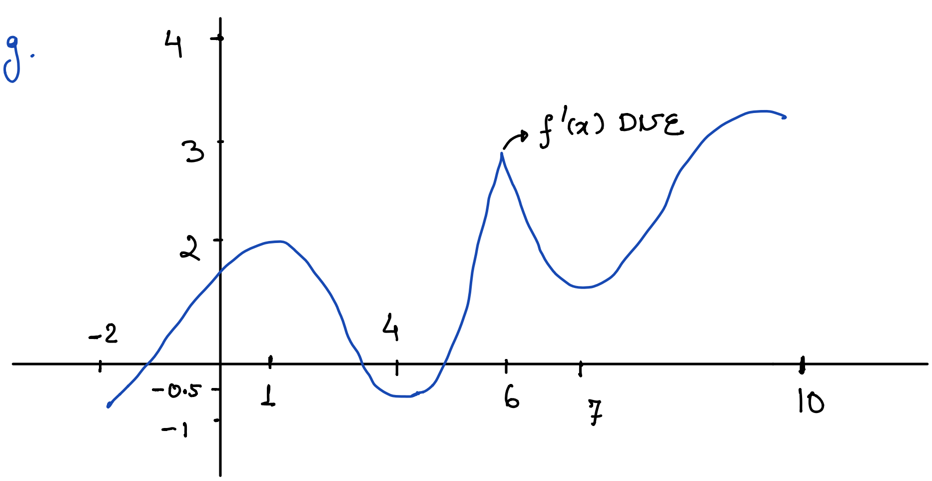
Relative Extrema (Maxima and Minima)

In this lecture we are going to learn about local maxima/minima and global maxima/minima.

Def^n: Let c be a point in the domain of $f(x)$. Then $f(x)$ is said to have

- a local maxima or local max at c with value $f(c)$ if $f(c) \geq f(x)$ for all x near c.
- a local minima or local min at c with value $f(c)$ if $f(c) \leq f(x)$ for all x near c.

e.g.



from the graph of a function given above, we see that it has a

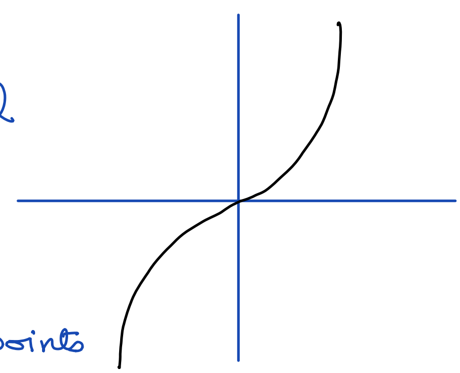
- local max at $x = 1$ w/ value 2
- local min at $x = 4$ w/ value -0.5
- local max at $x = 6$ w/ value 3
- local min at $x = 7$ w/ value 1.5

It looks from the graph above that local max/min are occuring at critical points.

However not every critical point is a local max or min.

e.g. If $f(x) = x^3$ then $f'(x) = 3x^2 \Rightarrow$ critical point is $x = 0$.

But $x = 0$ is neither a local max nor a local min.



So the critical points are the points where we will check for local max/min.

To check whether there is a local max/min we have the following test.

First derivative Test

Let c be a critical point of $f(x)$. Then

1) If $f'(x)$ goes from positive to negative at $x=c$ then $f(x)$ has a local max at $x=c$. In other words,

$f'(x) > 0$ for $x > c$ and $f'(x) < 0$ for $x < c$ $\Rightarrow$

local max at $x = c$.

2) If $f'(x)$ goes from negative to positive at $x=c$
$\Rightarrow$ local min at $x = c$.

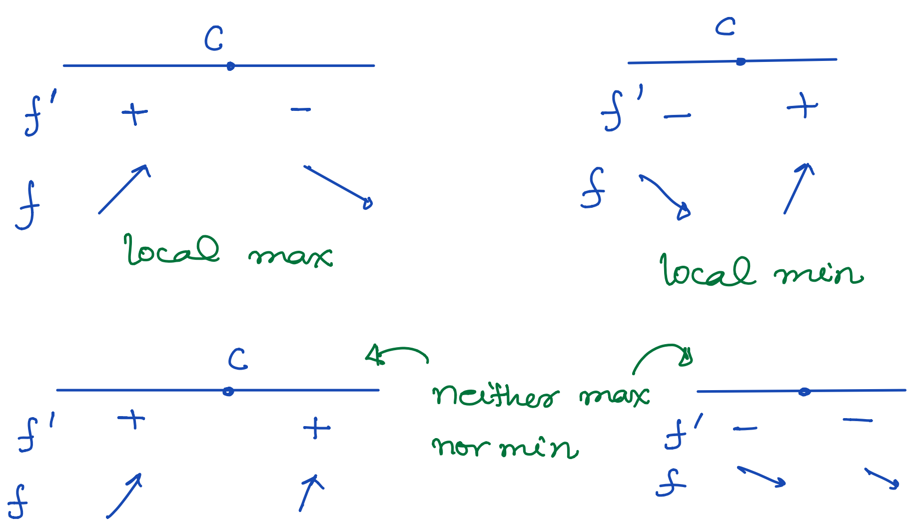
3) If the sign of $f'(x)$ remains the same, i.e.,

either $f'(x) > 0$ for $x > c$ and $x < c$

or $f'(x) < 0$ for $x > c$ and $x < c$

then $f(x)$ has a neither a local max nor a local min.

So you need to just remember



Thus, for example $f(x) = x^3$ gives $x=0$ as critical point and $f'(x)$ remains positive

$\Rightarrow$ $x=0$ is neither a local max or min. which is consistent with what we observe from the graph.

Remark :- Local extrema means local max and min.

Ques find local extrema for

① $f(x) = e^{x^2}$ ② $f(x) = 3x^3$

③ $f(x) = x^3 + 9x^2 - 21x + 36$.

Sol ① we find the critical points.

$f'(x) = e^{x^2} \cdot 2x = 0 \Rightarrow x=0$ is a critical point.

Now $f'(x) = e^{x^2} \cdot 2x < 0$ for $x < 0$

> 0 for $x > 0$

Thus, $f'(x)$ is going from negative to positive

$\Rightarrow$ $x=0$ is a local min with value $f(0) = e^{0^2} = 1$.

② $f'(x) = 9x^2 = 0 \Rightarrow x = 0$ is the only critical point. But $f'(x) > 0$ at $x < 0$ and $x > 0$

$\therefore$ $x = 0$ is neither a local max nor a local min and $f(x)$ has no local max/min.

③ $f'(x) = 3x^2 + 18x - 21 = 0 \Rightarrow 3(x^2 + 6x - 7) = 0$

$\Rightarrow 3(x+7)(x-1) = 0$

So $x = -7$ and $x = 1$ are the critical points.

	$f'(x)>0$		$f'(x)<0$	$f'(x)<0$		$f'(x)>0$
	$+$	-7	$-$	$-$	1	$+$
f	$\nearrow$		$\searrow$	f $\searrow$		$\nearrow$

So $f(x)$ has a local max at $x = -7$ with value $f(-7) = 281$

local min at $x = 1$ with value $f(1) = 25$.