M40: Ausgewahlte Kapitel der Mathematik: Topics in geometric analysis - Extrinsic geometric flows
Humboldt-Universität zu Berlin, Summer Semester 2023

Instructor:	Dr. Shubham Dwivedi Institut für Mathematik (Rud. 25) Room 1.309 dwivedis@math.hu-berlin.de
Website:	https://www.mathematik.hu-berlin.de/ dwivedis/EGFSS23.html
Moodle:	https://moodle.hu-berlin.de/course/view.php?id=118687 enrolment key: hamilton
Lectures:	Wednesdays 9:15 - 10:45 in Seminarraum 20 2.006 (Rud. 25) Thursdays 11:15 - 12:45 in Seminarraum 20 2.006 (Rud. 25)
Problem Sessions:	Wednesdays 11:15 - 12:45 in Seminarraum 30 4.007 (Rud. 25)
Language:	English

Short Description

Geometric flows are important tools in studying various geometric structures on a manifold. This course intends to be an introduction to the study of *extrinsic geometric flows*, i.e., to study the evolution of objects which lie in an ambient manifold, and to study many of its properties and applications. Important examples of such flows and the ones we plan to cover in the course are the **Curve shortening flow** and the **Mean curvature flow**. The target audience is advanced Bachelors and Masters's students and PhD students so only basic knowledge of Riemannian geometry and analysis (especially PDEs) will be very beneficial. A detailed (preliminary) discussion of topics is outlined below.

Topics to be covered

(1) Basics of Riemannian geometry and Ricci calculus with emphasis on calculations in local coordinates.
(2) Basics on Partial Differential Equations with a focus on parabolic PDEs; existence of solutions to such PDEs.
(3) Detailed study of the Heat equation and various PDE techniques to study it.
(4) Curve shortening flow.
(5) Theorems of Gage–Hamilton and Grayson.
(6) Monotonicity formulas and self-similar solutions.
(7) Introduction to the mean curvature flow.
(8) Evolution of geometric quantities along the MCF.
(9) Huisken's monotonicity formula and its applications.

The aforementioned topics are much more than what we'll actually be able to cover in the course.

Grading

The grades in the course will be decided either by an oral exam or by a presentation at the end of the semester. We'll decide the exact format once the course starts.

Literature

There are excellent introductions to the subject of curve shortening flow and the mean curvature flow and the materials presented in the class will be followed from the references mentioned below. In particular, [ACGL], [Zhu02], [CZ01] and [She] are good sources for self-study as well.

References

[ACGL] Ben Andrews, Bennett Chow, Christine Guenther, and Mat Langford, *Extrinsic geometric flows*, Graduate Studies in Mathematics, vol. 206, American Mathematical Society, Providence, RI, [2020] ©2020. MR4249616 ↑2

[CZ01] Kai-Seng Chou and Xi-Ping Zhu, *The curve shortening problem*, Chapman & Hall/CRC, Boca Raton, FL, 2001. MR1888641 ↑2

[She] Nick Sheridan, *Hamilton's ricci flow*, Bachlors's Thesis, available at https://www.maths.ed.ac.uk/~nsherida/ricciflow.pdf. ↑2

[Zhu02] Xi-Ping Zhu, *Lectures on mean curvature flows*, AMS/IP Studies in Advanced Mathematics, vol. 32, American Mathematical Society, Providence, RI; International Press, Somerville, MA, 2002. MR1931534 ↑2