Extrinsic Geometric Flows
Dr. Shubham Dwivedi

Humboldt-Universität zu Berlin
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Problem Set 4
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Problems

(1) Find a family of diffeomorphisms $y : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \times (-\infty, \infty) \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that the parametrization

$$(y,t) \mapsto (x(y,t), t - \log\cos(x(y,t)))$$

of the Grim Reaper satisfies the parametrized curve shortening flow equation.

(2) Show that the paperclip given implicitly by $\cosh y = e^{-t}\cos x$, and the hairclip given implicitly by $\sinh y = e^{-t}\cos x$ are solutions to the curve shortening flow.

(3) Recall the support function σ from Pset3 (4). Assume that for any embedded closed convex curve, we have the inequality

$$\int_{M^1} \sigma^2 ds \leq \frac{LA}{\pi}$$

where L is the length of the curve and A is the area of the interior. Prove that

$$\int_{M^1} \kappa^2 ds \geq \frac{\pi L}{A}.$$

(4) Prove that for any embedded closed convex curve evolving by curve shortening flow, the *isoperimetric ratio* $\dfrac{L^2}{A}$ is monotonically decreasing in time.