Extrinsic Geometric Flows
Dr. Shubham Dwivedi

Humboldt-Universität zu Berlin
Summer Semester 2023

Problem Set 3
Due date: 17.05.2023

Problems

(1) Let $X(t)$ be a CSF on $[0, T_{max})$. Suppose $\kappa(u,t) \leq K \ \forall \ (u,t) \in M^1 \times [0, T_{max})$. Prove the first order estimate from the lecture: If X' denotes the derivative with respect to some parameter x (for instance the arc length parameter s at time 0) then $\forall \ (u,t) \in M^1 \times [0, T_{max})$ we have
$$e^{-K^2 T_{max}} \leq |X'(u,t)| \leq C$$
where the constant $C < \infty$ depends on $|X'(u,0)|$.

(2) In the same situation as Prob. 1, compute the expression $N \cdot X^{(4)}$ from the lecture.

(3) Suppose the map $X(x,t)$ evolves by
$$\frac{\partial X}{\partial t}(u,t) = -\phi(u,t)N(u,t)$$
for some smooth function $\phi : M^1 \times [0,T) \to \mathbb{R}$. Prove that
$$\frac{\partial}{\partial t}\left(\frac{\partial}{\partial s}\right) = \phi\kappa\frac{\partial}{\partial s}.$$

(4) Let $X : S^1 \to \mathbb{R}^2$ be an immersed planar curve. The **support function** $\sigma : S^1 \to \mathbb{R}$ is defined as
$$\sigma(u) = \langle X(u), N(u)\rangle.$$
(a) Calculate $\int_{S^1} \sigma\kappa ds$.
(b) Prove that under CSF the evolution of the support function is
$$\sigma_t = \sigma_{ss} + \kappa^2\sigma - 2\kappa.$$